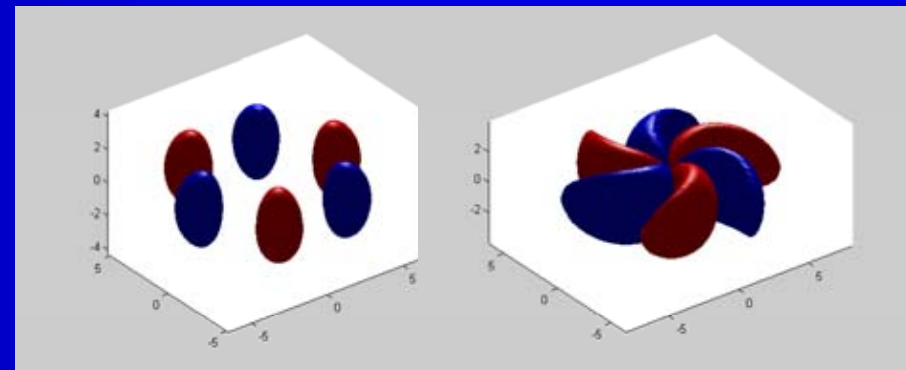
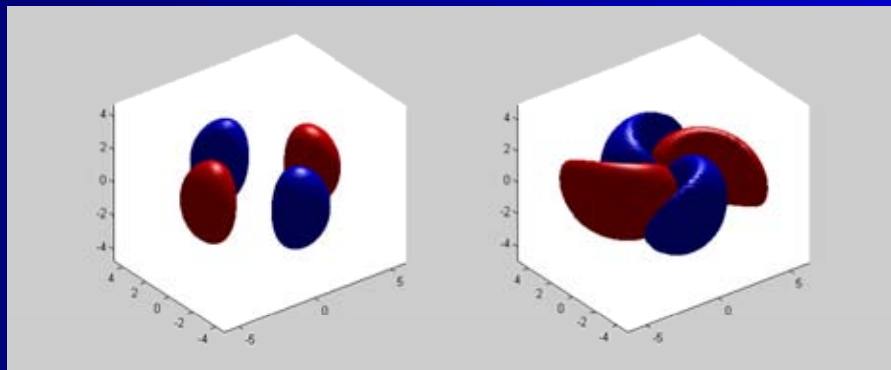
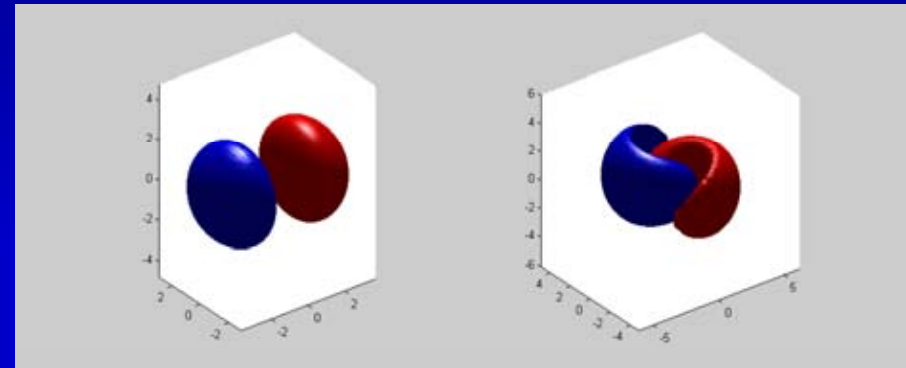
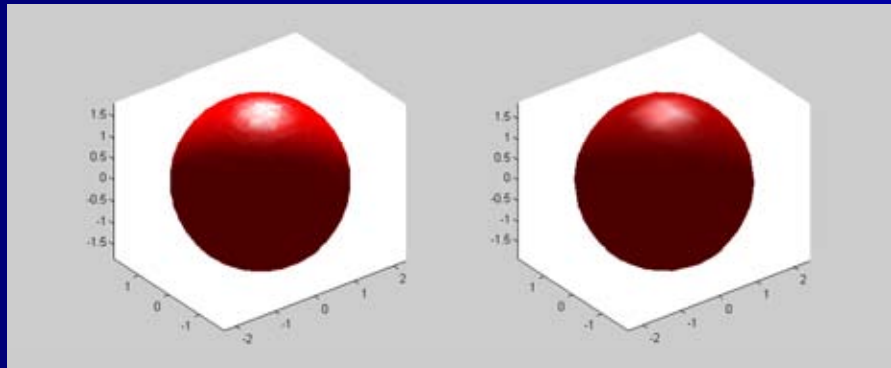
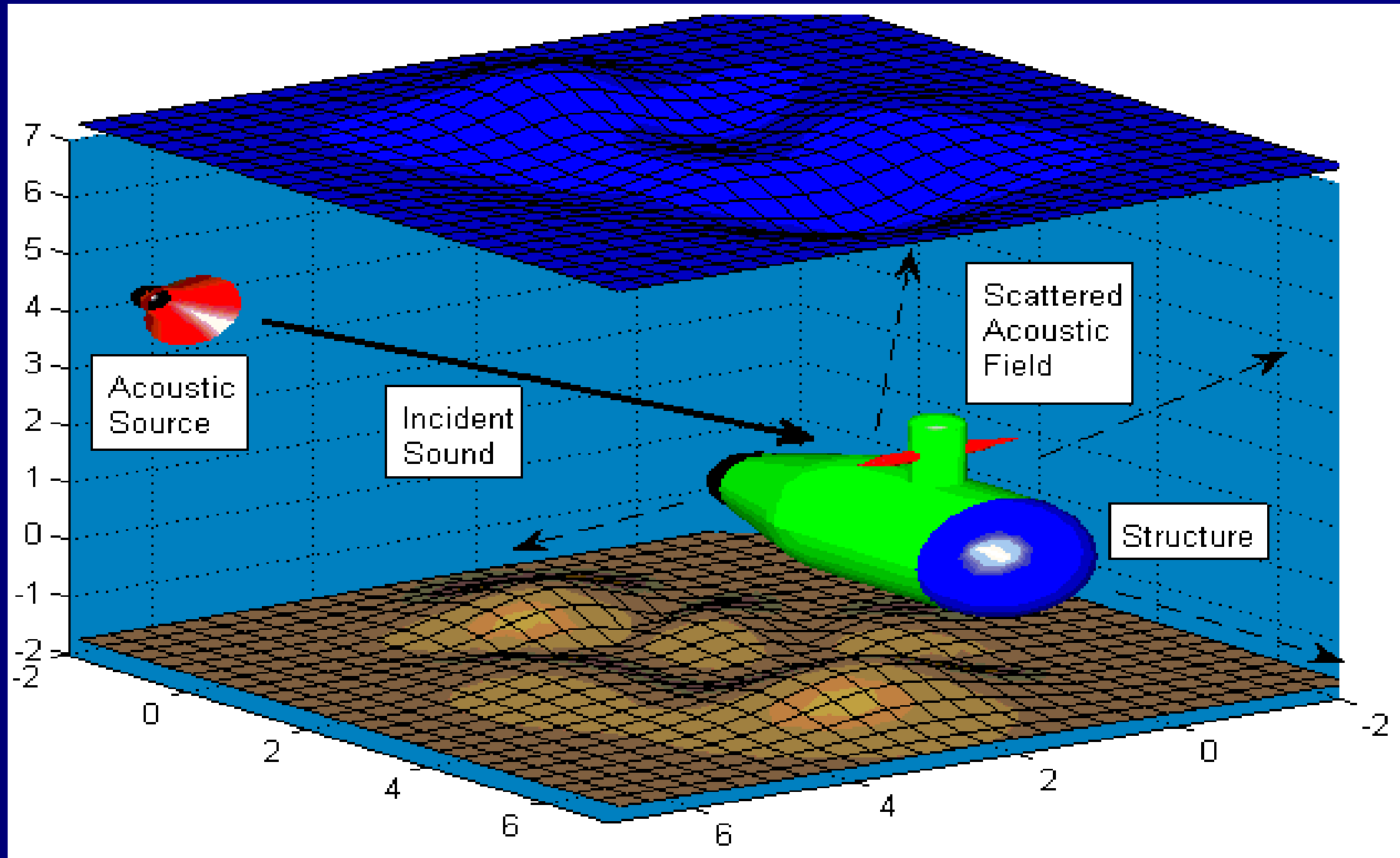


Implementing the Fast Multipole Boundary Element Method in Matlab

Daniel Wilkes, 12433805
Project Supervisor: Dr Alec Duncan

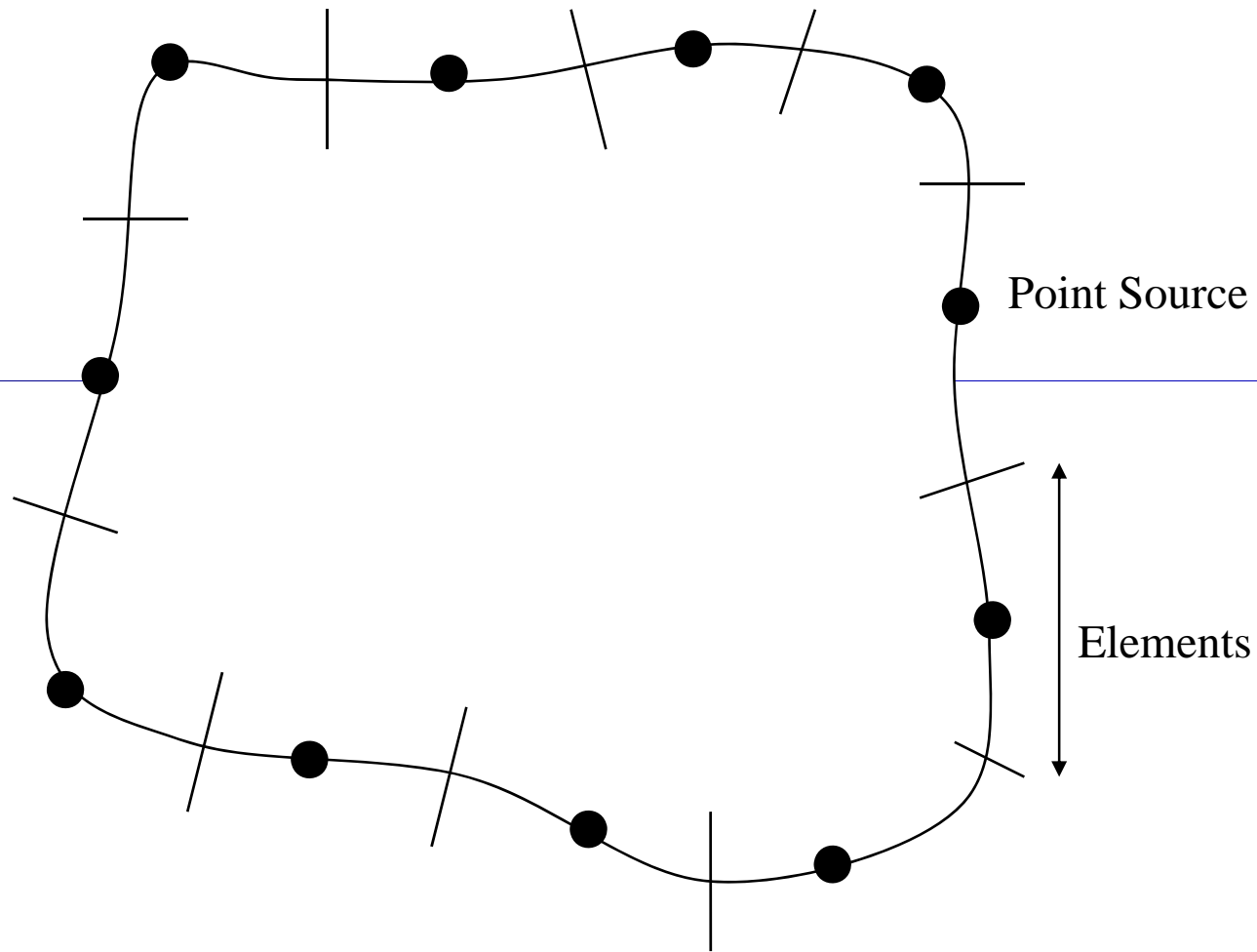


Coupled Fluid-Structure Interactions



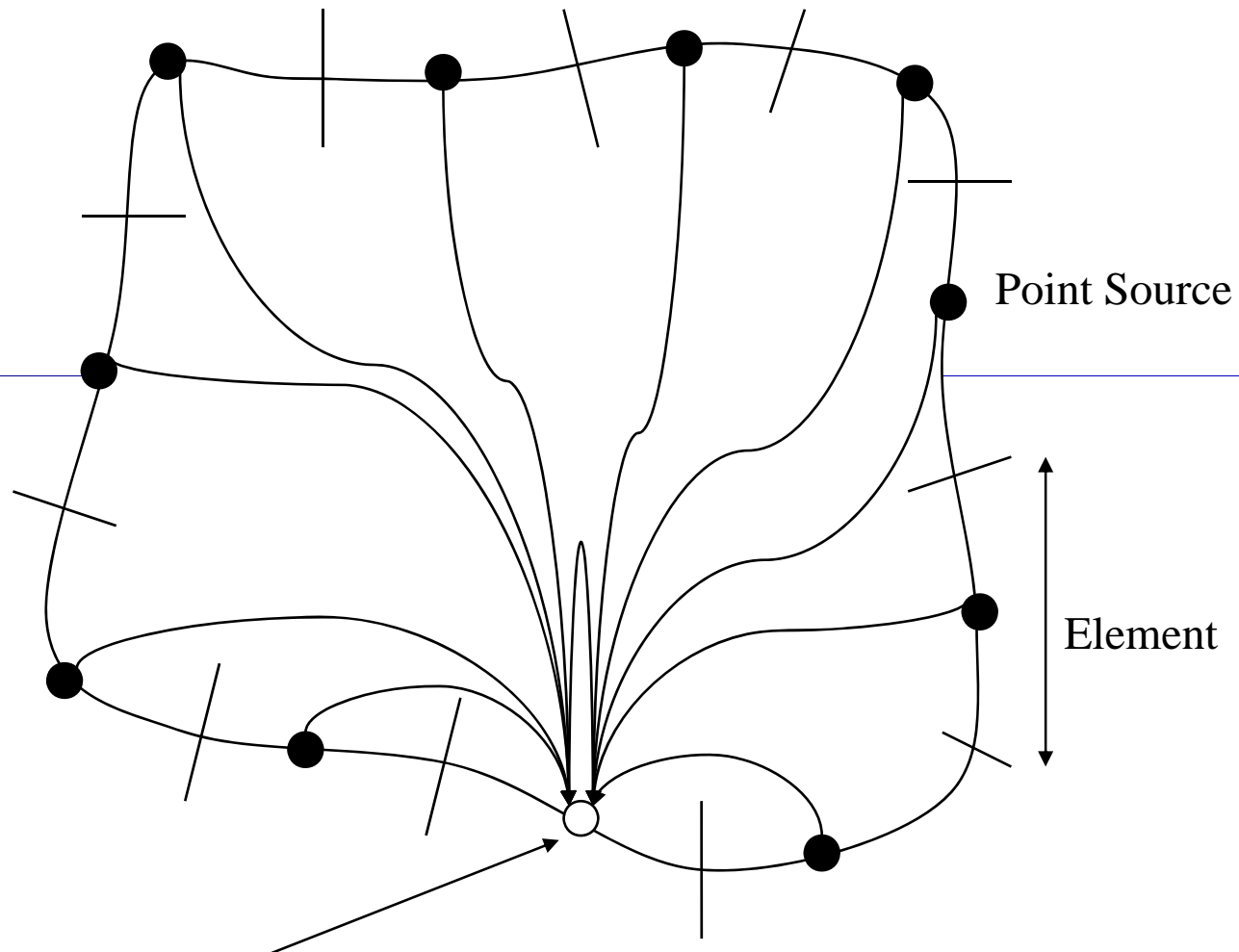
Sound Scattering from a Submerged Structure

The Boundary Element Method



Discretised Boundary for a 2D Object

The Boundary Element Method

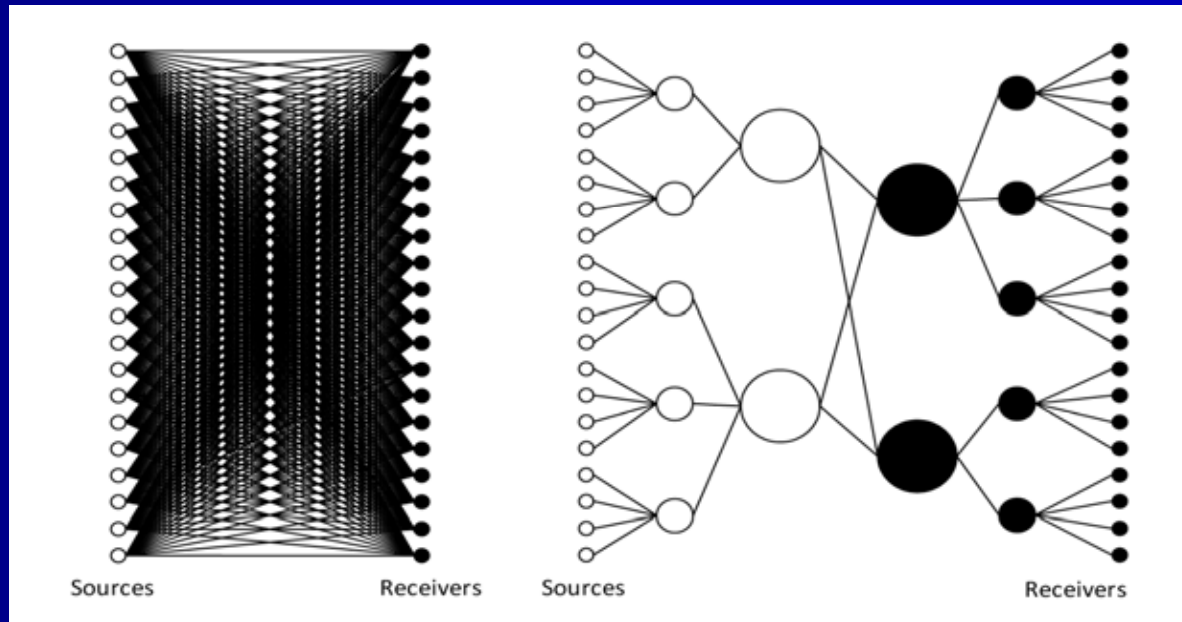


The acoustic pressure at *this* source is the sum of contributions from *all* sources

Discretised Boundary for a 2D Object

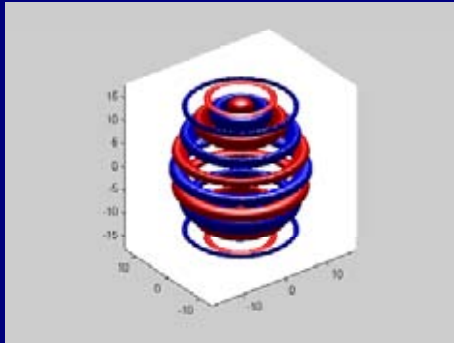
The FMBEM Concept

- Approximately calculate the matrix vector product in the BEM
- Equivalent to a summation of multipole sources at each receiver
- The S and R expansions can be used to represent multipoles with respect to local origins
- SS , RR and SR operations can be used to shift them to larger or smaller domains of validity

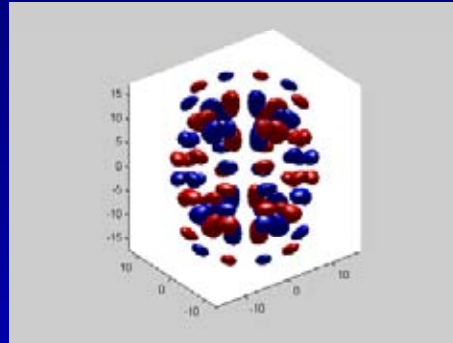


BEM Computation (left) and FMBEM computation (right) (adapted from Gumerov & Duraiswami, 2004)

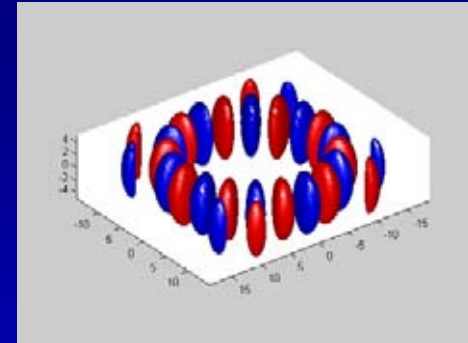
Singular and Regular Basis Functions



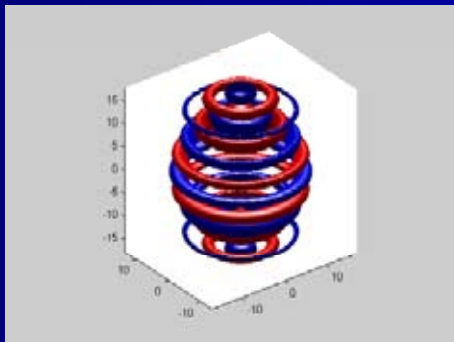
Re{R} $n = 10$ $m = 0$ isosurface



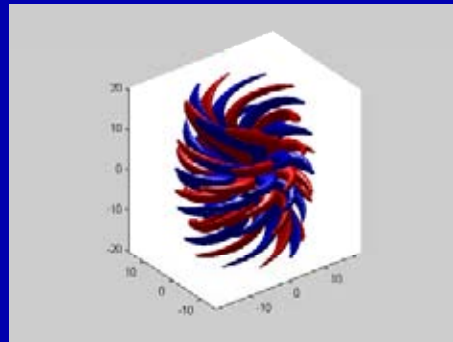
Re{R} $n = 10$ $m = 5$ isosurface



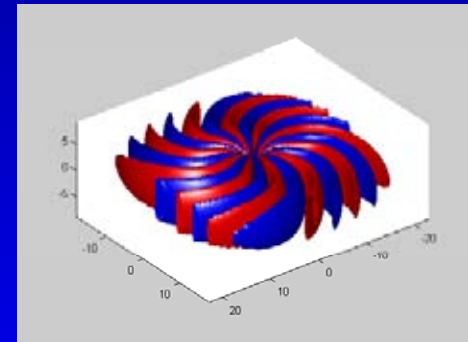
Re{R} $n = 10$ $m = 10$ isosurface



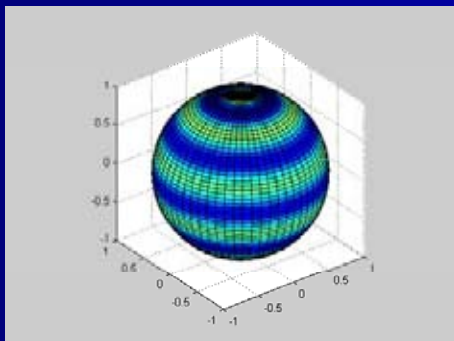
Re{S} $n = 10$ $m = 0$ isosurface



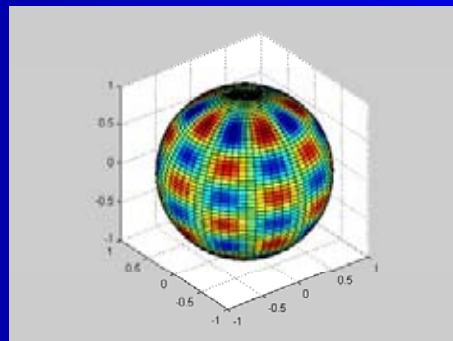
Re{S} $n = 10$ $m = 5$ isosurface



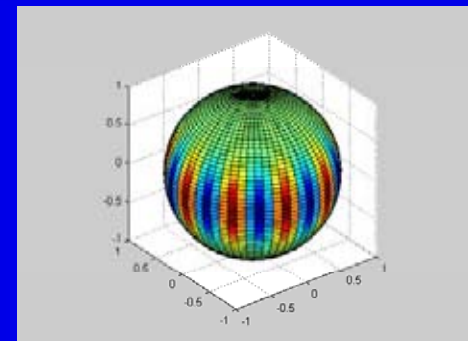
Re{S} $n = 10$ $m = 10$ isosurface



Y $n = 10$ $m = 0$

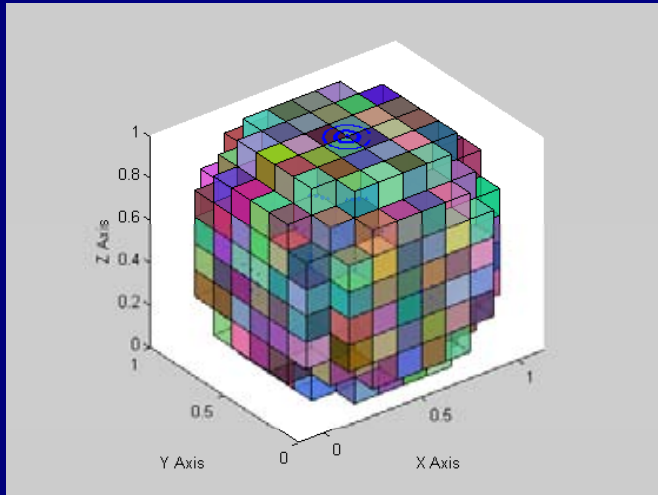


Y $n = 10$ $m = 5$

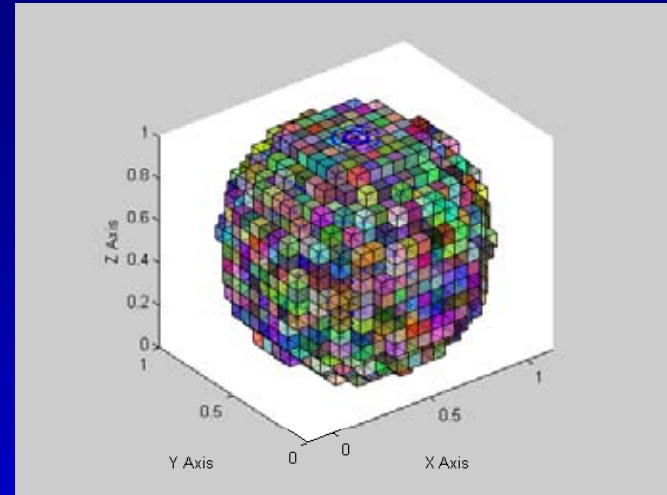


Y $n = 10$ $m = 10$

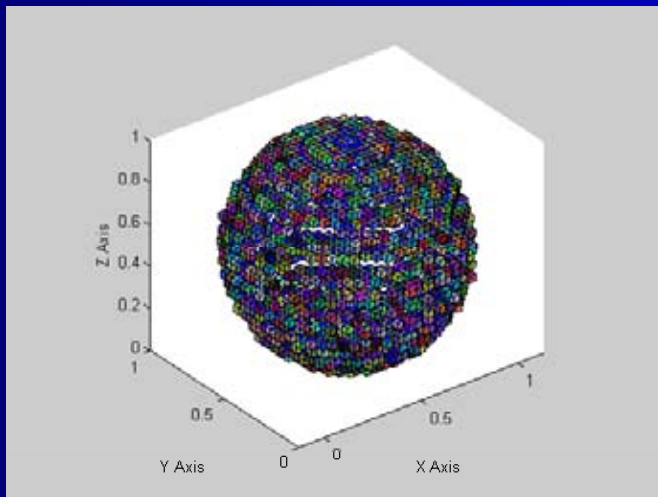
Boxes for Different Octree levels



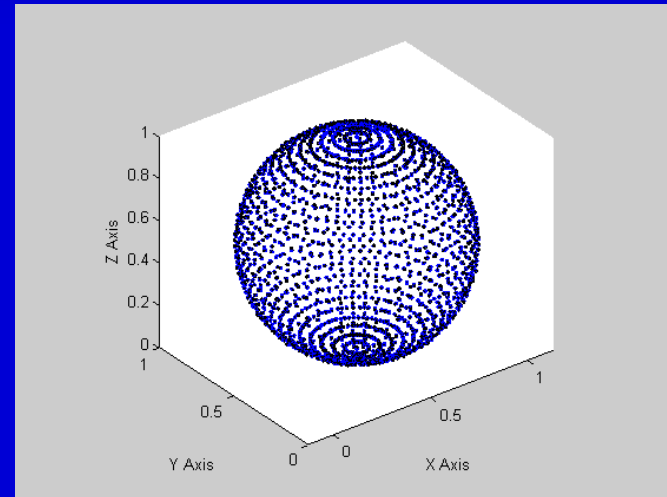
Clustering Parameter = 50, Octree Level = 3



Clustering Parameter = 10, Octree Level = 4

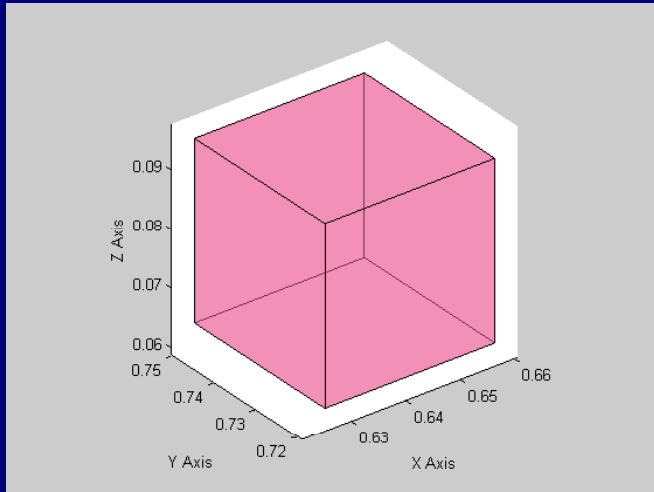


Clustering Parameter = 5, Octree Level = 5

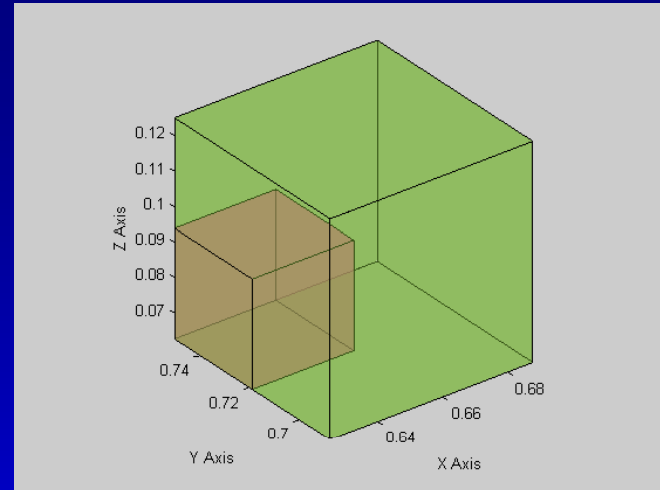


Clustering Parameter = 1, Octree Level = 7

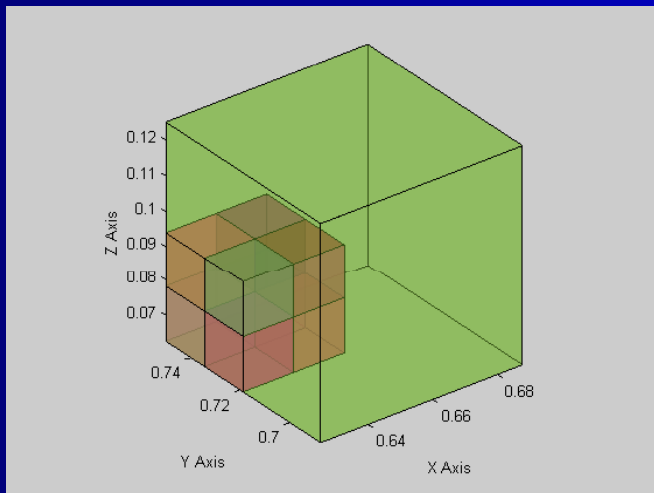
Octree Structure Searching



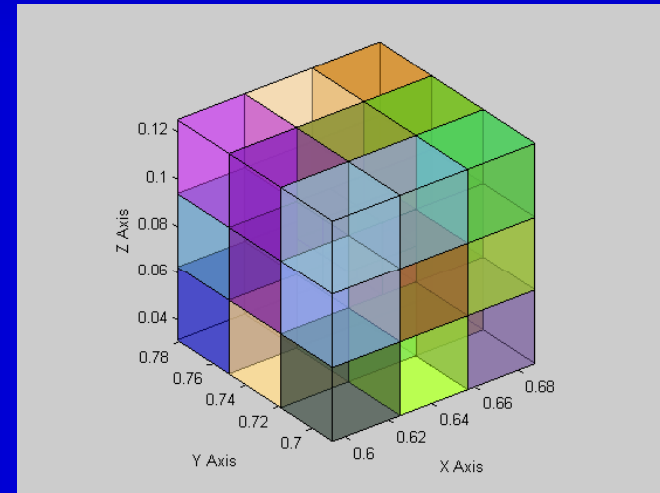
Box Number 24986, Octree Level = 5



Parent Box, Octree Level = 4



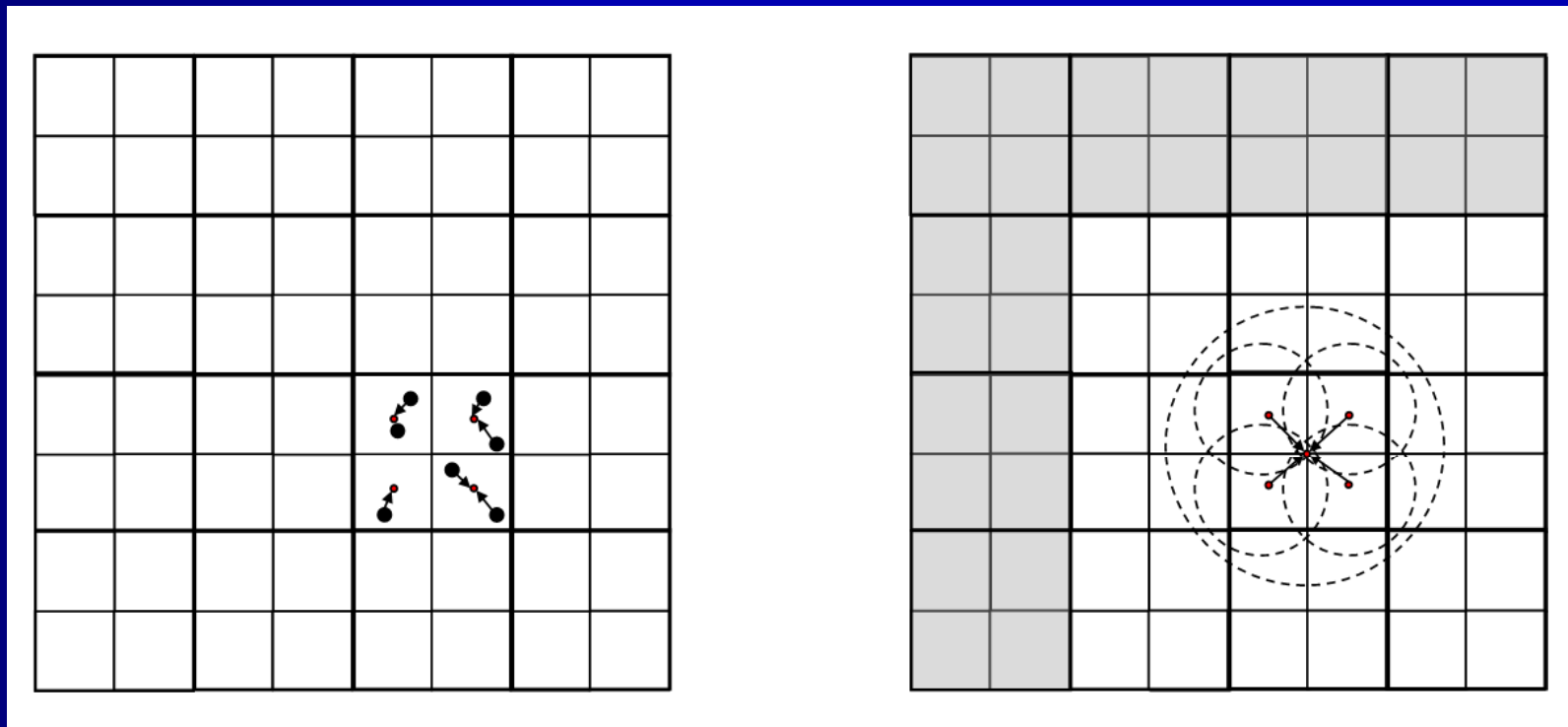
Children Boxes, Octree Level = 6



Neighbour Boxes, Octree Level = 5

Fast Multipole Procedure

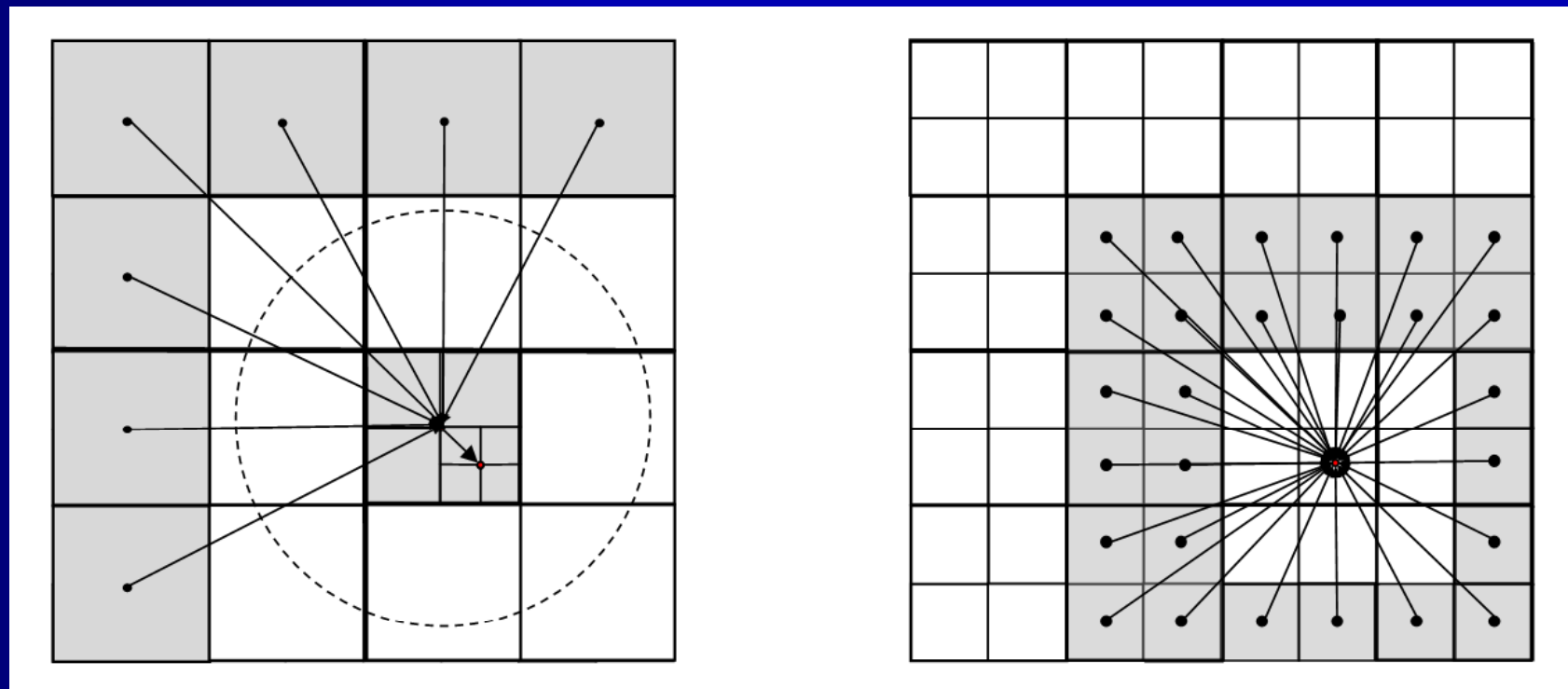
- Use the FMM to calculate the multipole interactions for the 'far' field
- Build S expansions for all sources wrt box centres and sum them
- SS translate each multipole sum to the parent box centre (up to level 2)



S Expansions and Summations (left) and SS Translations (right) (adapted from Gumerov & Duraiswami, 2004)

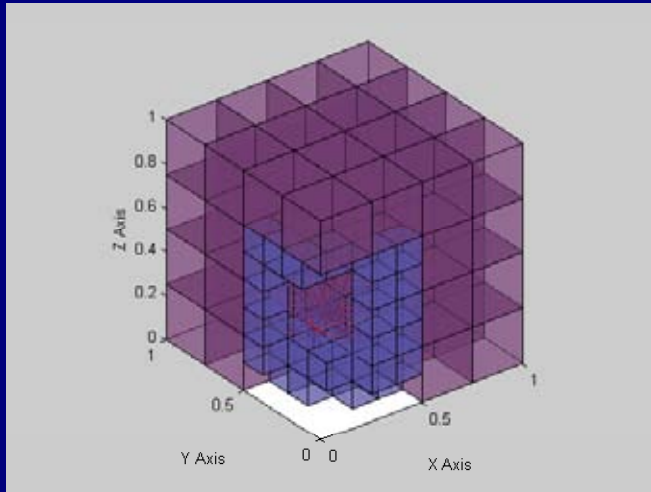
Fast Multipole Procedure

- At level 2 we have 64 S expansions (3 dimensions), 512 on level 3, etc
- These far field expansions can then be SR translated to the near field
- The SR translations are then summed and RR translated to the child boxes

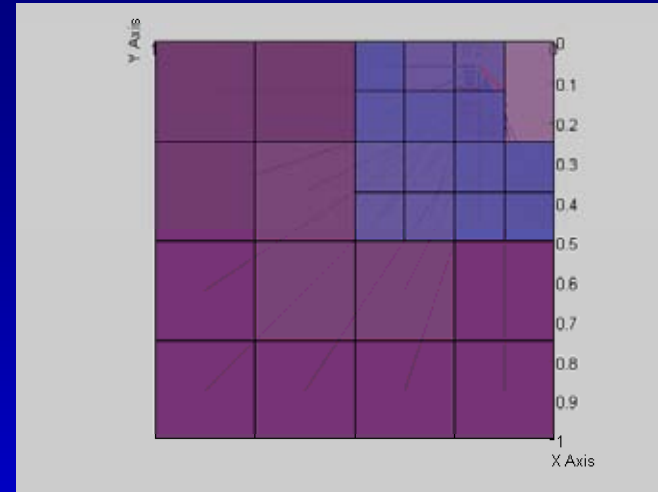


SR Translations for One Box at Level 2 and Level 3 (adapted from Gumerov & Duraiswami, 2004)

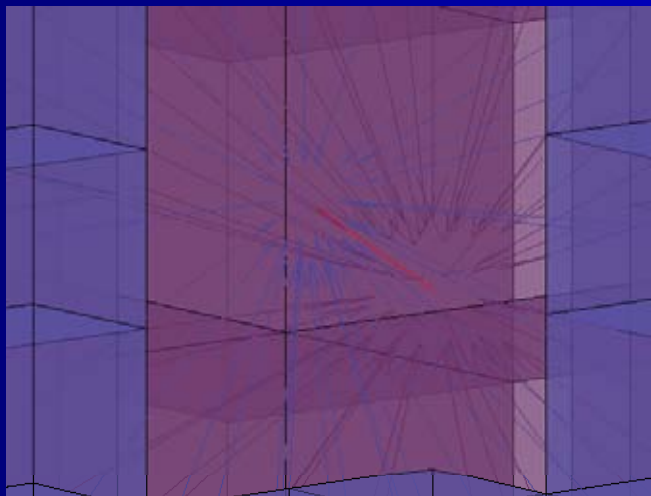
The Fast Multipole BEM in Matlab



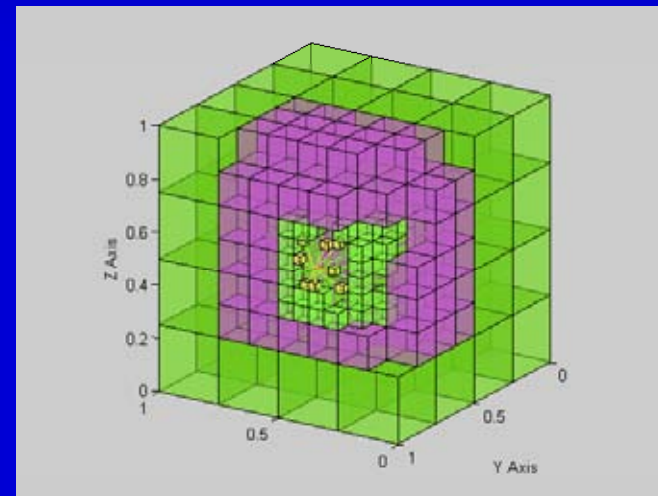
SR Translations for One Box on Source Octree Level



SR Translations for One Box on Source Octree Level

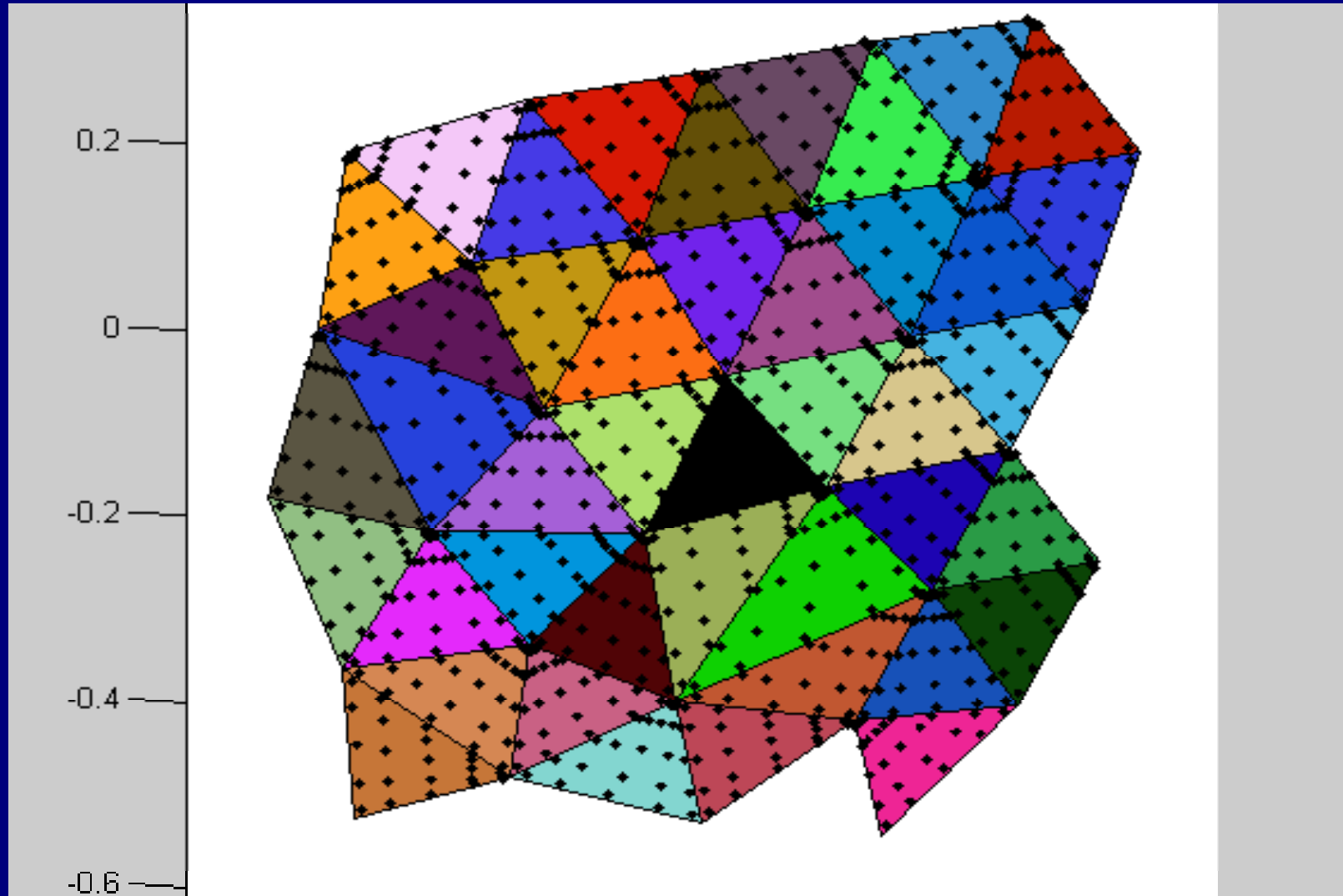


SR Translations for One Box on Source Octree Level



SR Translations for Source Box on 4 Level Octree

Numerical Integration: Gaussian Quadrature



Gaussian Quadrature of the Near Field Elements

The Fast Multipole BEM

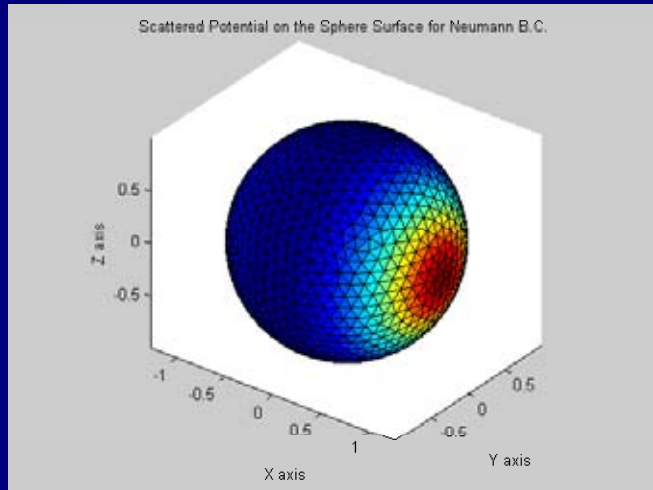
- Interactions of elements (Green's function integrated over the element)
- Also need to deal with the issues with BEM (non-uniqueness, singular integration)
- **Non-uniqueness of BEM solutions: Burton Miller formulation**
 - A linear combination of Green's 2nd identity and it's normal derivative has a unique solution at all frequencies. For complex potential ϕ

$$\pm \phi(y) = \int_S G(x, y) \frac{\partial \phi(x)}{\partial n(x)} - \frac{\partial G(x, y)}{\partial n(x)} \phi(x) dS(x) \quad (\text{Gumerov \& Duraiswami, 2009})$$

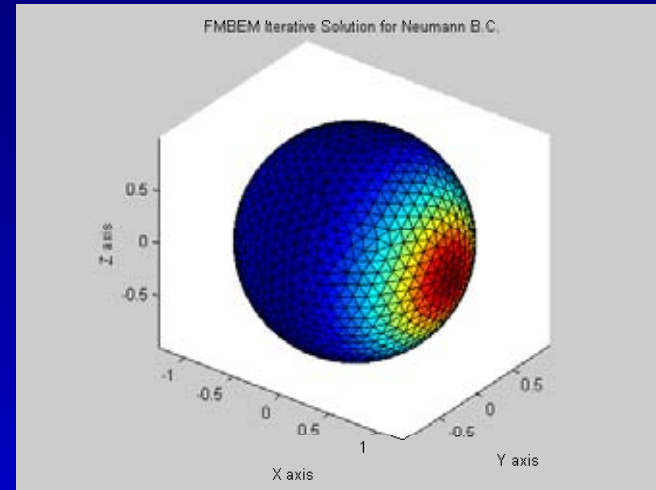
$$\pm \frac{\partial \phi(y)}{\partial n(y)} = \int_S \frac{\partial G(x, y)}{\partial n(y)} \frac{\partial \phi(x)}{\partial n(x)} - \frac{\partial^2 G(x, y)}{\partial n(x) \partial n(y)} \phi(x) dS(x)$$

- 4 coefficients to calculate
- Need to deal with singular and hyper-singular integrals

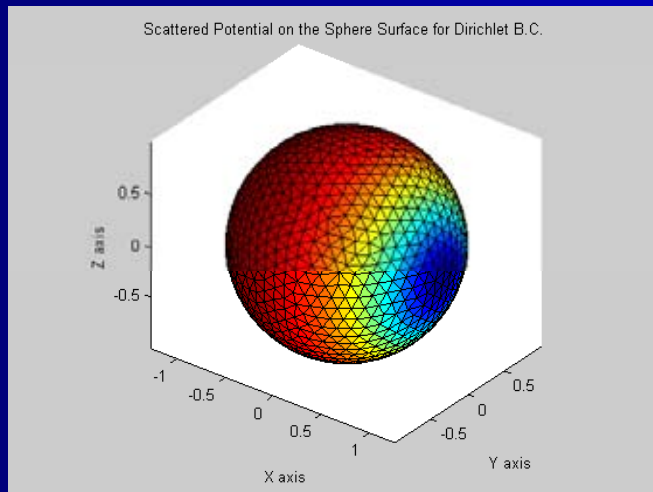
GMRES Iterative Solution for Sphere



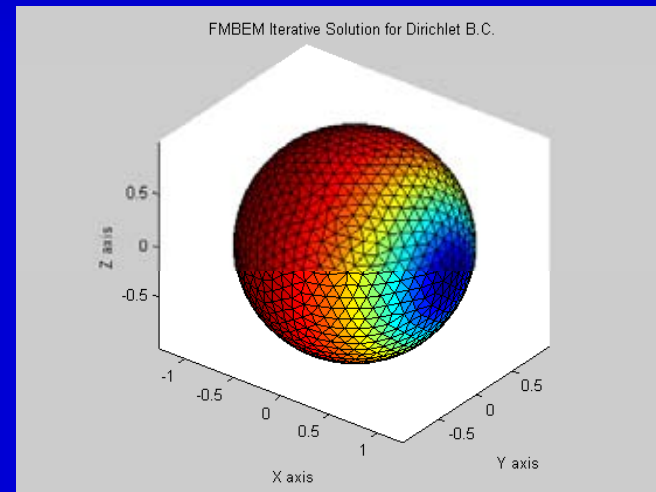
Unit strength Point Source at [2 0 0]



*5 GMRES Iterations for residual $5.1e-5$. 47.9 seconds
Relative residual norm = 0.0830*

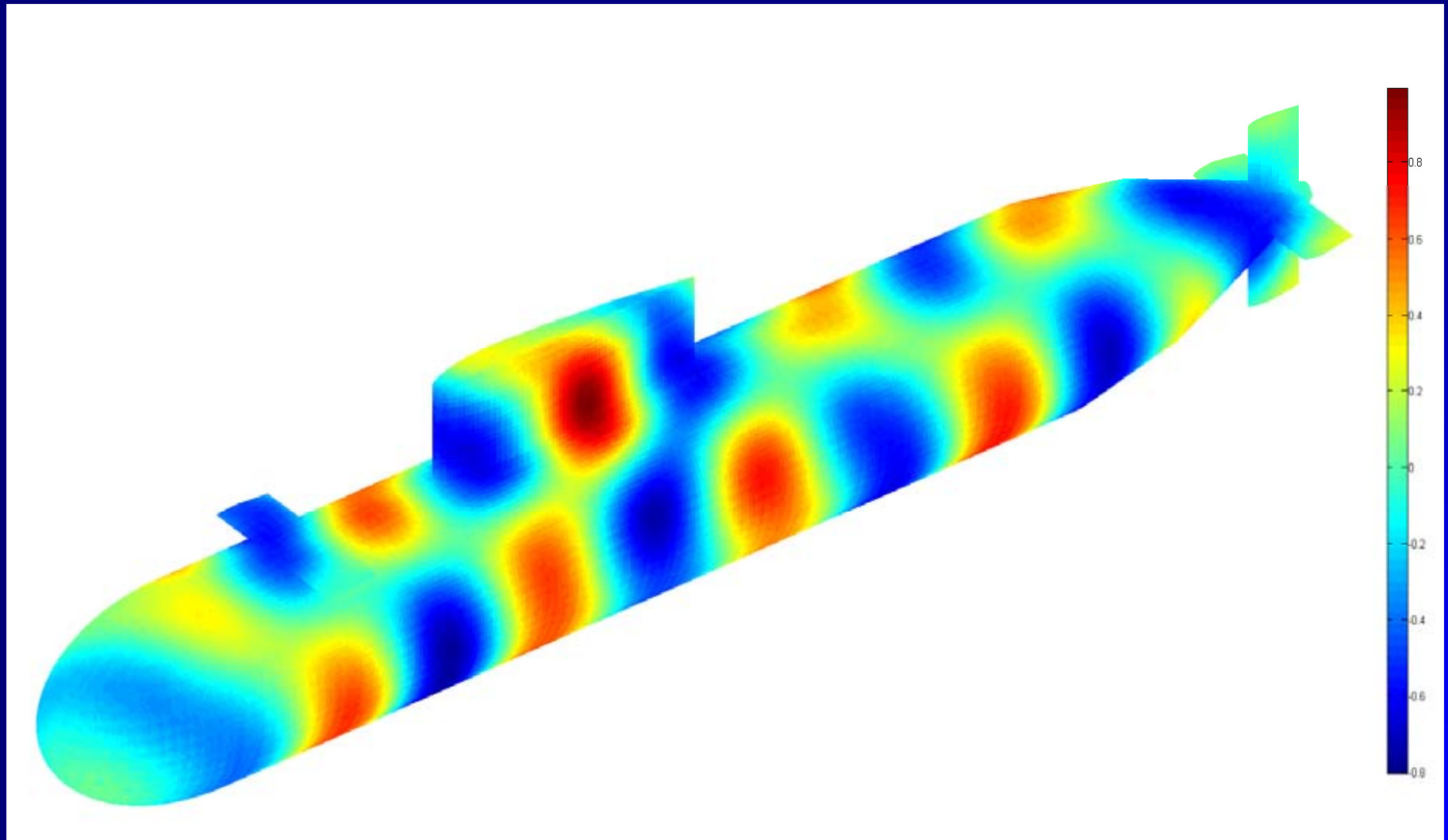


Unit strength Point Source at [2 0 0]



*9 GMRES Iterations for residual $9.8e-5$. 76.7seconds
Relative residual norm = 0.0577*

FMBEM for the BeTSSi Submarine Model



BeTSSi submarine model showing the real component of the complex far field pressure for the first BM term (involving the Green's function and pressure derivative normal to the surface).

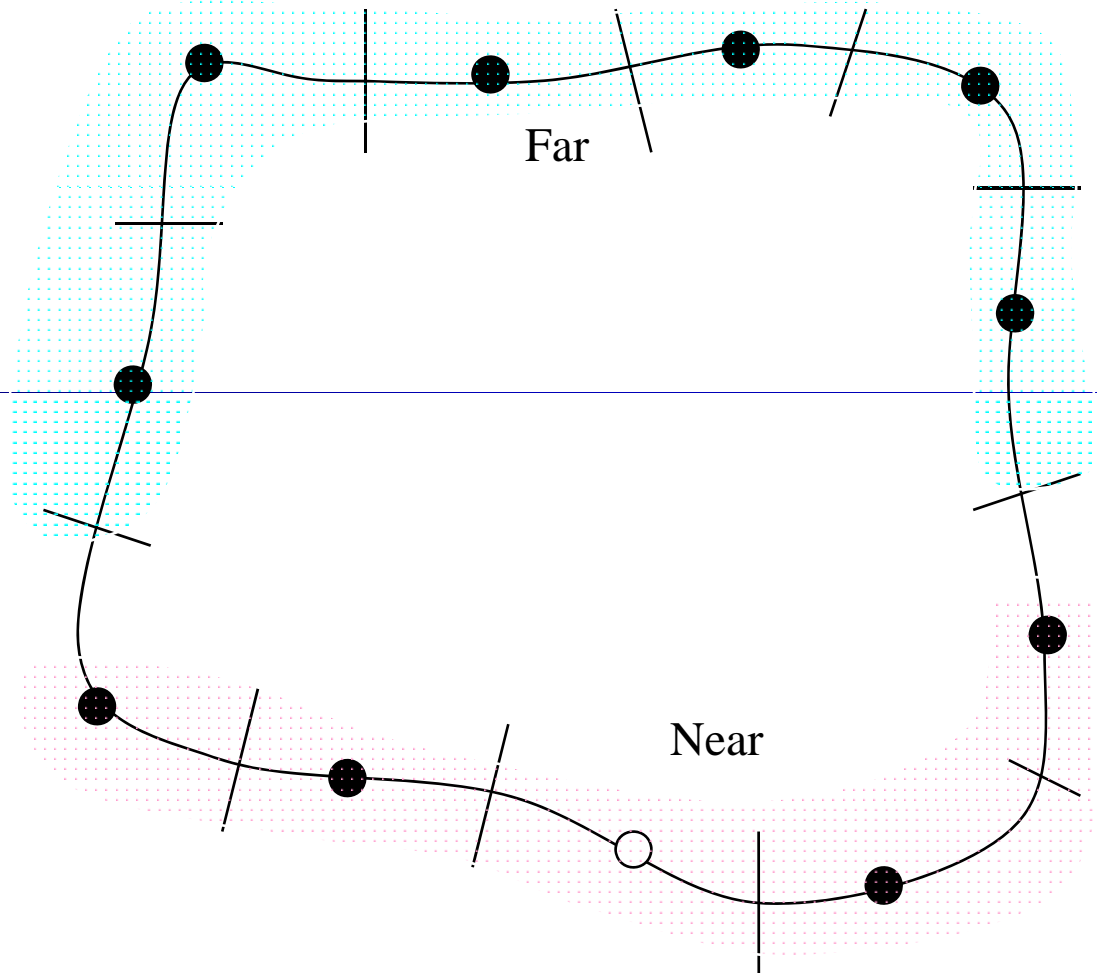
Further Work

- **Research Applications**
 - The FMBEM provides a fast method of calculating the exterior acoustic field of an arbitrary object using the Helmholtz equation
 - Only have pressure, normal velocity and impedance (mixed) BCs
 - For coupled fluid-structure interactions, a second model of the interior of the object is required
 - There are several papers on coupling the FMBEM to the finite element method for coupled interactions: Gaul et al 2009, Brunner et al 2009, Margonari & Bonnet 2005
 - The elastic wave equation can also be formulated as a boundary value problem and solved with FMM (Chaillat et al, 2009). This has been applied to piecewise homogeneous domains for $N \log N$ complexity
- We plan to investigate methods to couple a FMBEM acoustic model to a FMBEM elastic wave model for coupled fluid-structure interactions

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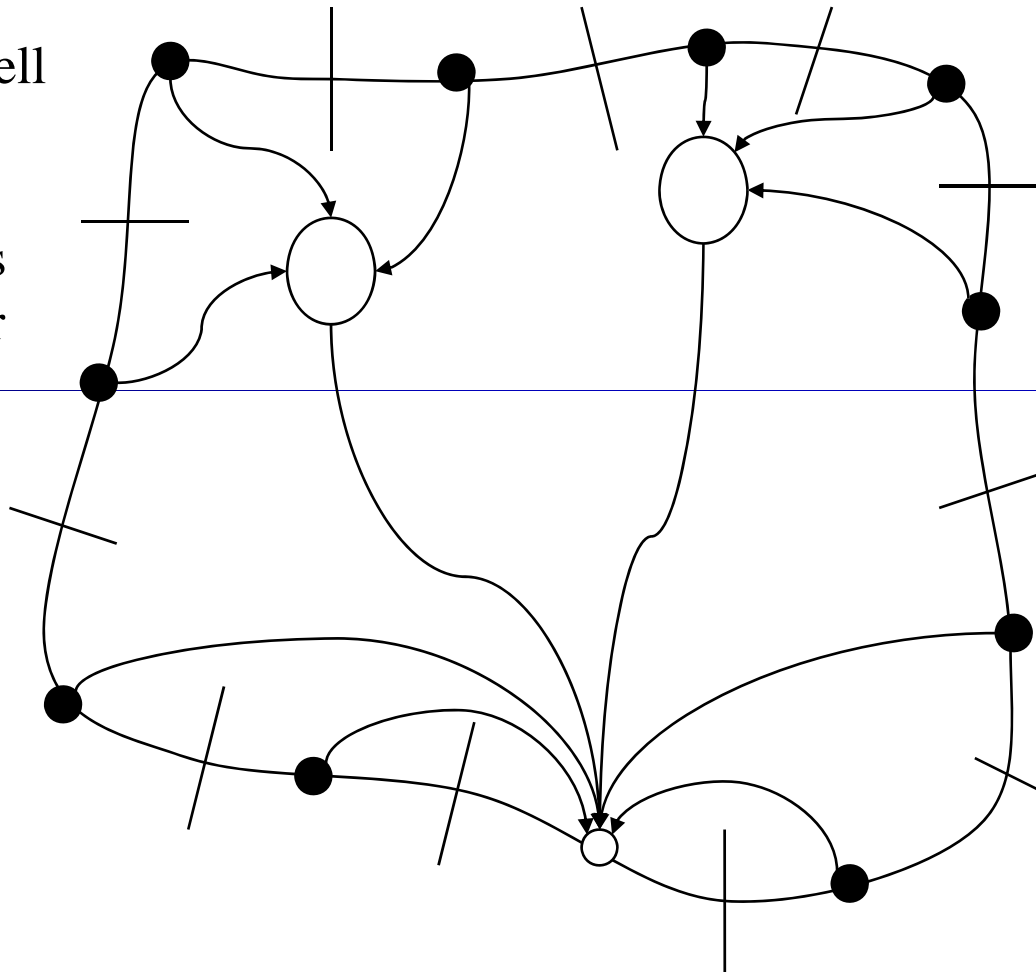
The Fast Multipole BEM



Near and Far Field Sources with respect to a Source Point

The Fast Multipole BEM

Include well separated sources as multipoles at receiver



Multipole Summations for Far Sources

Near and Far Field Domain: The Octree Structure

- Need to determine what is 'near' and 'far' from a particular source
- Binary octree structure splits domain into cubes
- Normalize mesh to $[0\ 1] \times [0\ 1] \times [0\ 1]$ domain
- Create bit-interleaved binary representations of the node positions
- These become the box numbers of each source

Bit from x co-ordinate: 1
Bit from y co-ordinate: 0
Bit from z co-ordinate: 1

Result: $101 = 5$

